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QUASI-ONE-DIMENSIONAL MODEL OF THE ROD–TARGET INTERACTION

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Numerical techniques are proposed for determining the integral characteristics of penetration of a rod into an target. An algorithm for solving two-dimensional elastoplastic problems is employed. To construct the solution, a one-dimensional finite-element column is used (a two-dimensional domain is replaced by a one-dimensional domain).

Introduction. Rod–target interaction is accompanied by complex physical processes. An exact mathematical formulation of the problem that takes into account the physicomechanical properties of the rod and target materials (including their dependence on temperature and deformation rate) and the physics of phenomena associated with the penetration of the rod into the target (for example, phase transitions in the material due to the strong heating) leads to a complicated nonlinear system of differential equations that can be solved only with the use of high-speed computers with large random-access memory. The problem of high-velocity interaction between a rod and a target belongs to the class of contact problems of dynamic deformation of inelastic bodies. Rather effective algorithms for the numerical solution of these problems are currently available (see, e.g., [1–7]). However, in many cases, it is difficult to estimate the errors that arise inevitably in the numerical solution. When the problem is solved in Lagrangian coordinates, one has to repeatedly regenerate the grid in the regions of considerable distortion of the cells and to extrapolate the solution from the previous grid to the new grid, which leads to loss of accuracy. When Eulerian coordinates are used, errors arise, in particular, in calculating rapidly changing contact and free surfaces and in averaging quantities in the case of substance overflow from one cell to another. In the formulation and solution of problems of material deformation under intense thermal and force actions, one should bear in mind that experimental data on physicomechanical properties of materials under extreme conditions are usually incomplete. The constants of a material, which characterize its strength properties, and the constitutive equations are determined with an error that can reach dozens of percent. Therefore, it is of practical interest to construct a sequence of mathematical models whose complexity increases with the capability of providing an adequate description of quantitative and qualitative characteristics of the impact.

Sagomonyan [8] gave a detailed review of one- and two-dimensional models of high-velocity impact.

One of the first one-dimensional models of high-velocity impact is the hydrodynamic model proposed by Lavrent'ev for estimating the penetration depth of a cylindrical rod into a semi-infinite target [9]. According to this model, we have $(l/l_0)/(\rho_t/\rho_r)^{-1/2} = \text{const}$, where l_0 is the initial length of the rod, l is the penetration depth, and ρ_t and ρ_r are the densities of the target and rod materials, respectively.

Various modifications of the hydrodynamic model have been proposed to incorporate approximately the effect of the strength properties of the rod and target materials. For example, in the model developed under the assumption that the target failure follows the "force a plug out" mechanism, only the effect of shear stresses is taken into account among all factors responsible for the penetration resistance of the target. Tate [10] proposed a modification of the hydrodynamic theory that takes into account the strength of rod and

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target and gave experimental data on the dynamic yield stress. All the approximate models include "fitting" coefficients, which can be determined experimentally.

Below, we consider two numerical techniques of determining the integral characteristics of the impact, which we call quasi-one-dimensional techniques. These techniques are based on the algorithms for solving two-dimensional dynamic problems of elastoplastic deformation [4–7]; at the same time, a one-dimensional finite-element column is used to obtain the solution (a two-dimensional domain is replaced by a one-dimensional domain).

1. Constitutive Equation. In deriving the constitutive equation, we assume that $1+\gamma'_{ij} \simeq 1$, where γ'_{ij} are the components of the elastic strain tensor. This assumption is valid for most metals. Moreover, the volumetric deformation of an element of the medium is assumed to be elastic. Under these assumptions, the kinematic relations, valid for arbitrary strains, can be written in the form [11]

$$\dot\gamma_{ij} = \alpha e'_{ij} - \gamma'_{jk}\omega_{ki} - \gamma'_{ik}\omega_{kj} - \varphi'_{ij}, \quad \dot\gamma = \alpha e, \quad \rho_{\rm t} = \rho_{\rm r}\alpha^{3/2}, \quad \alpha = 1 - 2\gamma/3,$$

$$e_{ij} = e^{\rm el}_{ij} + \varphi_{ij}, \qquad \varphi_{ij}\delta_{ij} = 0, \qquad e'_{ij} = e_{ij} - e\delta_{ij}/3, \tag{1.1}$$

$$\gamma'_{ij} = \gamma_{ij} - \gamma\delta_{ij}/3, \qquad e = e_{ij}\delta_{ij}, \quad \gamma = \gamma_{ij}\delta_{ij},$$

where φ_{ij} are the components of the plastic strain-rate tensor, and $\omega_{ij} = \partial v_i/\partial x_j - \partial v_j/\partial x_i$ are the components of the rotation-rate tensor.

The total strains ε_{ij} satisfy the relations

$$\dot\varepsilon_{ij} = \varepsilon_{ki}\frac{\partial v_k}{\partial x_j} + \varepsilon_{kj}\frac{\partial v_k}{\partial x_i} = e_{ij}.$$

To obtain constitutive relations between the stresses, strains, and temperatures, we assume that the internal energy U is a function of the first and second invariants of the elastic strain tensor and entropy S: $U = U(\gamma, J_2, S)$, where $J_2 = \gamma'_{ij}\gamma'_{ij}/2$. Using the law of conservation of energy $\sigma_{ij}e_{ij} = \rho_{\rm t}\dot U$ and relations (1.1), we obtain $\sigma e + \sigma'_{ij}e'_{ij} = \rho_{\rm t}\alpha e U_\gamma + \rho_{\rm t}\gamma'_{ij}(\alpha e'_{ij} - \gamma'_{ki}\omega_{kj} - \gamma'_{kj}\omega_{ki} - \varphi'_{ij})U_{J_2} + \rho_{\rm t}U_S\dot S$. Hence,

$$\sigma = \rho_{\rm t}\alpha U_\gamma, \qquad \sigma'_{ij} = \rho_{\rm t}\alpha\gamma'_{ij}U_{J_2}, \qquad \rho_{\rm t}U_S\dot S = \rho_{\rm t}\gamma'_{ij}\varphi'_{ij}U_{J_2}. \tag{1.2}$$

We rewrite the last relation in (1.2) in the form

$$T\dot S = \frac{1}{\alpha\rho_{\rm t}}\sigma'_{ij}\varphi'_{ij}, \tag{1.3}$$

where $T = U_S$ is the temperature. Relations (1.2) and (1.3) are expressed in terms of the free energy $\Phi = U - ST$ as follows:

$$\sigma = \rho_{\rm t}\alpha\Phi_\gamma, \quad \sigma'_{ij} = \rho_{\rm t}\alpha\gamma'_{ij}\Phi_{J_2}, \quad -T\,\Phi_{TT}\dot T = \frac{1}{\alpha\rho_{\rm t}}\sigma'_{ij}\varphi'_{ij} + T\Phi_{\gamma T}\dot\gamma + T\Phi_{J_2T}\dot J_2. \tag{1.4}$$

The quantity $-T\Phi_{TT}$ has the meaning of heat capacity for constant strains and is denoted by c_V.

Further elaboration of Eqs. (1.4) depends on the choice of the function Φ and on the assumed character of irreversible plastic deformation.

If we assume that the heat capacity c_V is independent of γ and J_2, the average stress σ does not depend on J_2, and the relation between σ'_{ij} and γ'_{ij} is linear, the function Φ can be written in the form

$$\Phi = [\psi(\gamma) + T\varphi(\gamma) + 2\mu J_2 + f(T)]/\rho_{\rm r},$$

where μ is the shear modulus. In this case, relations (1.4) become

$$\sigma = \alpha^{5/2}(\psi_\gamma + T\varphi_\gamma), \quad \sigma'_{ij} = 2\mu\alpha^{2/5}\gamma'_{ij}, \quad c_V\dot T = (1/\alpha\rho_{\rm t})\sigma'_{ij}\varphi'_{ij} + (\alpha/\rho_{\rm r})eT\varphi_\gamma. \tag{1.5}$$

In the elastoplastic model, we assume the following relation between the average stress σ, the invariant of elastic strains γ, and the temperature T:

$$\sigma = K\alpha^{5/2}\gamma - c_V\Gamma\rho_{\rm t}(T - T_0). \tag{1.6}$$

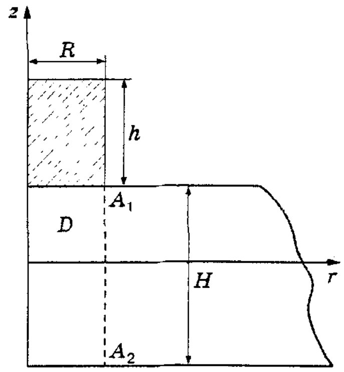

Fig. 1

Here $K = E/(3(1-\nu))$ is the coefficient of volumetric expansion, E is Young's modulus, ν is Poisson's ratio, Γ is the Grüneisen coefficient, T is the absolute temperature, and T_0 is the temperature corresponding to normal conditions (usually 300 K).

Relation (1.6) follows from the first relation in (1.5) provided that

$$\psi = K\gamma^2/2 + (3/2)c_V T_0 \rho_{\rm r} \ln \alpha, \qquad \varphi_\gamma = -(c_V/\alpha)\Gamma\rho_{\rm r}.$$

In this case, the third relation in (1.5) takes the form

$$\dot{T} = \sigma'_{ij}\varphi'_{ij}/(c_V\alpha\rho_{\rm t}) - \Gamma T e. \tag{1.7}$$

The rate of irreversible deformation is determined by the relations

$$\varphi'_{ij} = \lambda\gamma'_{ij}. \tag{1.8}$$

The procedure for calculating the nonnegative multiplier λ is described in [7]. Equations (1.1) and (1.6)–(1.8) determine the elastoplastic model.

In the numerical algorithms considered below, the calculation domain consists of one finite-element column subject to boundary conditions that model the flow of the substance. In the calculation domain, we use the model of elastoplastic flow with large strains (1.6)–(1.8). The stresses and strains in the calculation domain are determined by the algorithm developed in [4–7].

The techniques differ in the form of boundary conditions at the calculation-domain boundary, formulated according to which qualitative and quantitative characteristics of the impact are to be determined.

2. Quasi-One-Dimensional Model for Estimating the Limit Penetration Depth of the Rod into the Target. The rod is considered as an undeformable cylinder of radius R, height h, and mass M. The target of thickness H is an isotropic elastoplastic medium. We consider the domain D (calculation domain) located under the rod and bounded by the lateral surface A_1A_2 (Fig. 1).

In the impact, the time-dependent shear σ_{rz} and normal σ_r stresses occur at the lateral surface A_1A_2. To analyze the stress-strain state, one has to determine these stresses as functions of the spatial coordinates and time. Therefore, a two-dimensional problem should be solved. In the quasi-one-dimensional model, the interaction between the domain D and the remaining part of the target is modeled by boundary conditions at A_1A_2.

To estimate the limit penetration depth, we formulate the conditions at A_1A_2 as follows. The radial stress σ_r is assumed to vanish. Obviously, of all possible variants of the boundary conditions for σ_r, this condition ensures minimum penetration resistance of the target. If there are no radial stresses on the lateral surface A_1A_2, radial velocities occur, which lead to the outflow of a part of the material from the calculation domain. The material that flows out through the surface A_1A_2 is not taken into consideration in further calculations. Since the mass of this material has a nonzero axial velocity, its elimination in the calculations means that some part of the momentum in the axial direction is ignored, which leads to deceleration of the rod.

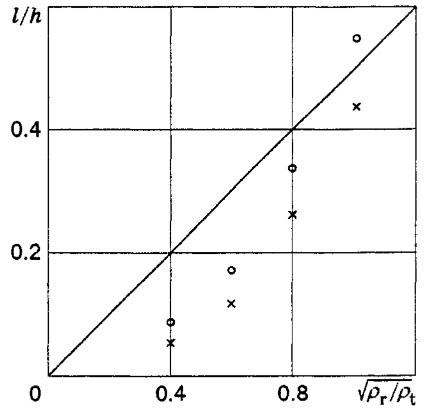

Fig. 2

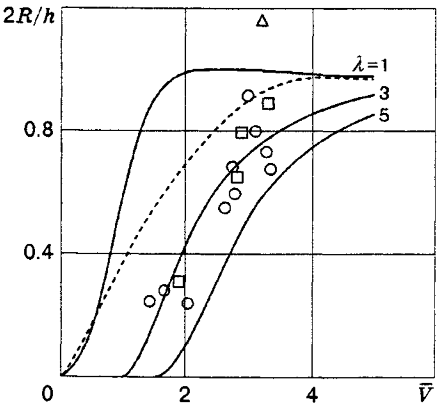

Fig. 3

The boundary conditions for the axial velocity u_z and the shear stress σ_{rz} at the surface A_1A_2 are formulated as follows. First, u_z is assumed to vanish at the lateral surface. If $|\sigma_{rz}| < \tau^*$ in this case (τ^* being the yield point in shear or the shear resistance), the solution is determined for the condition $u_z = 0$ at A_1A_2. If $|\sigma_{rz}| \geqslant \tau^*$, the solution is found for $|\sigma_{rz}| = \pm\tau^*$; in this case, the sign of σ_{rz} is chosen the same as that of σ_{rz} for a zero value of u_z. If the solution is determined by the techniques described in [4–7], no iterations are required to satisfy the above-formulated boundary conditions.

3. Quasi-One-Dimensional Model for Estimating the Limit Depth of the Rear Spalling Zone. One of the main factors that determine rear spallings is the pressure amplitude at the shock-wave front. Obviously, the pressure amplitude is maximum if it is assumed that the rod does not deform during penetration and, of all variants of the boundary conditions for the radial components of velocity and stress, the condition $u_r = 0$ is satisfied on the surface A_1A_2.

The boundary condition for shear stress σ_{rz} is taken to be the same as in Sec. 2.

4. Dependence of the Penetration Depth on the Ratio of Densities of the Rod and Target Materials. The rod is assumed to be an undeformable cylinder of mass M that has an initial impact velocity V_0. Then its velocity is set equal to the normal velocity at the upper boundary of the finite element of the calculation domain which is in contact with the cylinder base. Since the calculation domain consists of one finite-element column, the solution is calculated with the use of the algorithm given in [4–7] by finite formulas.

The input data of the problem were as follows: the initial velocity $V_0 = 5 \cdot 10^4$ cm/sec, the rod radius $R = 0.5$ cm, the rod height $h = 1.0$ cm, the density of the target material $\rho_t = 8 \cdot 10^3$ g/cm^3, $\rho_t/\rho_r = 0.4$, 0.6, 0.8, and 1.0 (ρ_r is the density of the rod density), the Young's modulus $E = 2 \cdot 10^5$ MPa, the tangent modulus $E' = 0$, the Poisson's ratio $\nu = 0.3$, the yield point in shear $\tau_s = 5 \cdot 10^2$ MPa, the Grüneisen coefficient $\Gamma = 2.0$, the heat capacity $c_V = 8.96 \cdot 10^2$ J/(kg · K), $T_0 = 300$ K, the target thickness $H = 5$ cm, and the number of finite elements in the column $N = 50$.

Figure 2 shows the quantity l/h (l is the penetration depth of the rod) as a function of $\sqrt{\rho_r/\rho_t}$. The crosses and circles refer to the results obtained under the assumptions that the radial velocity and radial stress vanish at the boundary of the calculation domain, respectively. The solid curve refers to the Lavrent'ev hydrodynamic model. For the aforementioned input data, both variants of boundary conditions at the boundary of the calculation domain give close results.

5. Dependence of the Depth of Penetration of a Cylindrical Rod into a Target on the Impact Velocity. Tate [10] presented experimental data on the high-velocity impact of cylindrical rods of mild steel against a target of the same material and compared them with theoretical results obtained with the use of the modified hydrodynamic model. The yield point was $Y_p = 11$ kbar. The diameter of specimens was 0.6 cm. The modified hydrodynamic model includes a dimensionless parameter λ, which characterizes the strength of the rod and target. This parameter is determined from the Hugoniot adiabat and ultimate strength.

Figure 3 shows the penetration depth versus the rod velocity $\bar{V} = \rho_t V_0^2/(4\sigma_T)$. The triangles, squares, and circles correspond to the experimental data of [10]. The triangles, squares, and circles refer to the rod length $h = 1.25$, 1.5, and 2.5 cm, respectively. The solid curves refer to the results obtained by the modified hydrodynamic model [10] and the dashed curve to calculations by the technique described in Sec. 2. The theoretical curve agrees satisfactorily with the experimental data with allowance for their scatter.

The above examples show that the simple calculation techniques proposed here may be used to estimate the integral characteristics of the impact processes.

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